Acoustic Receptivity Measurements Using Modal Decomposition of a Modified Orr–Sommerfeld Equation

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Boundary-layer receptivity to acoustic disturbances plays a key role in transition from laminar to turbulent flow. Acoustic disturbances interact with strong streamwise gradients at the leading edge to create Tollmien–Schlichting waves in the boundary layer. Measurement of TS receptivity to downstream-traveling sound is complicated by the presence of Stokes waves in the boundary layer and upstream-traveling acoustic reflections that also generate downstream-traveling TS waves. Active noise control is used to cancel reflections and enables the measurement of boundary-layer receptivity to downstream-traveling sound. TS wave amplitudes are extracted from hotwire data using biorthogonal decomposition of modified Orr–Sommerfeld equations that include acoustic disturbance solutions. The new method is implemented to measure the acoustic receptivity of a 20:1 modified-super-ellipse leading edge on a flat plate. The data yield acoustic receptivity coefficients consistent with the results of previous computational studies. For comparison, pulsed-sound measurements are also obtained which are comparable to both biorthogonal decomposition and direct numerical simulation results. The new measurement technique provides for efficient decomposition of measurements and sets the stage for renewed experimental effort in this area.

Nomenclature

\[ A \]
Left modified Orr–Sommerfeld matrix operator

\[ \tilde{A} \]
Augmented left modified Orr–Sommerfeld matrix operator

\[ B \]
Right modified Orr–Sommerfeld matrix operator

\[ \tilde{B} \]
Augmented right modified Orr–Sommerfeld matrix operator

\[ C_{St} \]
Complex Stokes wave amplitude

\[ C_{TS} \]
Complex Tollmien–Schlichting wave amplitude

\[ D \]
Partial derivative operator with respect to wall-normal coordinate

\[ \hat{\epsilon} \]
Weighted residual of modal reconstruction

\[ F \]
\[ 2\pi f \nu / U_\infty^2, \]
Nondimensional frequency

\[ f \]
Forcing frequency (Hz)

\[ H \]
\[ \delta^*/\theta, \]
Boundary-layer shape factor
Understanding and controlling boundary-layer transition from laminar to turbulent flow is critical for numerous aerodynamics applications. For example, the reduction in skin friction drag for laminar flow compared to turbulent flow could provide a substantial savings in fuel and weight for a commercial airliner. Despite its importance, transition is still not fully understood.

Because of the open-flow nature of boundary layers, transition is initiated when irrotational (acoustic) and rotational (turbulent) disturbances interact with a surface and create boundary-layer fluctuations. This process, called receptivity, sets the initial amplitudes of disturbances which grow and eventually lead to turbulent flow. Receptivity has been a focus of the boundary-layer stability community for over 40 years, but studying the process of how disturbances become entrained in the boundary layer remains challenging, especially in experiments. Understanding the receptivity process will further our ability to predict and delay boundary-layer transition.

The receptivity of planar, two-dimensional sound has been addressed by theory, experiments, and direct numerical simulation (DNS). Theoretical studies utilizing triple deck theory and the unsteady-
boundary-layer equations showed that the interaction of freestream disturbances with surface features generate Tollmien–Schlichting (TS) waves. These surface features include the leading edge,\textsuperscript{2} localized (e.g., 2D discrete roughness)\textsuperscript{4} and non-localized roughness,\textsuperscript{5} and strong streamwise gradients.\textsuperscript{6}

TS waves are a solution to the Orr–Sommerfeld (OS) equation:

\[
A \frac{\partial \phi}{\partial y} = B \phi
\]

which governs the stability of small disturbances in a parallel basic state. The small disturbances are contained in the vector \( \phi \), where:

\[
\phi = \left[ \hat{u}, \frac{\partial \hat{u}}{\partial y}, \hat{v}, \hat{p} \right]^T
\]

A and B are 4 x 4 matrix operators shown in the Appendix, variables with hats denote variables in Fourier space, and superscript \( T \) indicates the transpose operator. For the spatial problem, the OS equation is an eigenvalue problem for the complex streamwise wavenumber \( \alpha \). If the imaginary portion of \( \alpha \) is less than zero, the mode is spatially unstable and grows as it travels downstream. In this paper, only two-dimensional disturbances with \( w' = 0 \) and \( \partial / \partial z = 0 \) are considered.

TS waves are unstable over a range of Reynolds numbers. Moving downstream at a constant nondimensional frequency, \( F = 2 \pi f \nu / U^2_{\infty} \), the Reynolds number where instability first occurs is the branch I Reynolds number. The Reynolds number where the TS wave restabilizes is the branch II Reynolds number. Between branches I and II, TS waves grow exponentially as they travel downstream. During this growth, nonlinear interactions may occur, which eventually cause the boundary layer to transition to turbulence. Understanding how acoustic fluctuations become entrained in the boundary layer and create TS waves will be a fundamental building block in understanding the transition process.

Experiments at Arizona State University (ASU),\textsuperscript{7–10} coupled with matching DNS studies,\textsuperscript{11–14} investigated acoustic receptivity by introducing planar freestream sound into the test environment and measuring the amplitude of TS waves near branch II. Early experiments used continuous acoustic forcing,\textsuperscript{7} while later experiments used pulsed-sound forcing.\textsuperscript{8, 10}

The presence of Stokes waves (acoustic boundary-layer disturbances at zero angle of incidence) presents a challenge while measuring boundary-layer receptivity to acoustic disturbances. In the case of continuous acoustic forcing, a hotwire placed in the boundary layer will detect a combination of TS waves, Stokes waves, and sting vibration effects. Because all three phenomena occur at the forcing frequency, using a narrow band-pass filter will not separate the TS wave from contamination. Several separation techniques have been developed to measure TS wave amplitudes, but each method has drawbacks. Continuous forcing creates measurement issues and contaminates the experiment through complex duct acoustics and wake resonance.\textsuperscript{8} Pulsed forcing requires a large number of ensemble averages and has poor frequency resolution.\textsuperscript{10}

A. Complex Plane Spiral Analysis

Saric et al.\textsuperscript{7} utilized a complex plane analysis technique to measure receptivity coefficients. Because the acoustic wavelength is orders of magnitude larger than the TS wavelength, the Stokes wave has nearly a constant phase over one TS wavelength. To exploit this, hotwire measurements are taken at several streamwise locations across one TS wavelength and the real and imaginary components of the velocity signal in Fourier space are plotted in the complex plane. These data form a spiral whose radius is the growing amplitude of the TS wave. The contributions due to sting vibration and the Stokes wave are constants that simply offset the spiral from the origin.

There are two principal drawbacks to this method. First, large numbers of measurements are needed to obtain just one TS wave measurement. To measure one receptivity coefficient requires boundary-layer scans at multiple streamwise locations.\textsuperscript{8} Scanning only in the streamwise direction, as opposed to taking multiple boundary-layer scans, is not feasible because the wall-location is not known a priori and the TS-wave peak is a strong function of \( y \). Second, continuous acoustic forcing creates a complex acoustic field in the test section. A portion of the freestream acoustic wave is reflected by the change of cross-sectional area at the diffuser and 90° corner downstream of the test section. The analyses of Heinrich & Kerschen\textsuperscript{3} and Hammerton & Kerschen\textsuperscript{15} suggest that the receptivity to upstream-traveling waves is greater than downstream-traveling waves, so the presence of the reflected waves can affect the receptivity measurement. Saric & White\textsuperscript{8} also noted a global circulation effect in the test section during continuous forcing. This effect, which may be
caused by wake resonance described by Takagi and Konishi,\textsuperscript{16} may artificially enhance certain bands of TS waves. Overall, the complex plane technique is tedious and suffers from multiple difficulties associated with continuous acoustic forcing.

B. Pulsed-Sound Technique

Saric & White\textsuperscript{8} and White et al.\textsuperscript{10} use a pulsed-sound technique to separate Stokes and TS waves. The pulsed-sound technique utilizes the differences in group velocity of the two waves; the Stokes wave travels at the speed of sound, while the TS wave travels downstream at a fraction of the freestream velocity. Sound is emitted from speakers upstream of the model in a short burst (two to four cycles) and this sound interacts with the leading edge to create a packet of TS waves. The Stokes waves rapidly pass the measurement location and, after a short delay, the TS waves pass the measurement location. Data acquisition is timed such that the boundary-layer hotwire acquires data once the Stokes waves have passed. Large numbers of ensemble averages are used to account for random noise in the signal and the receptivity coefficient is calculated in the frequency domain through the separate averaging of real and imaginary parts.

Although this technique removes the effects of Stokes waves, the large number of ensemble averages required is cumbersome. Obtaining one boundary-layer profile requires many hours of measurement.\textsuperscript{10} Another drawback of this technique is the lack of frequency resolution. The length of the wave packet is constrained by the timing of the Stokes and TS waves which limits the frequency resolution of this technique. Although the pulsed-sound method avoids the wake resonance problem, diffuser reflections are present and may still contaminate the measurements. Furthermore, there are significant differences between the pulsed-sound results of Saric & White\textsuperscript{8} and White et al.\textsuperscript{10} Specifically, the receptivity coefficient appears to be a strong function of freestream velocity. Due to the small number of experimental measurements, it is not clear if this trend is physical or an artifact of the pulsed-sound method.

C. Present Objectives

Previous acoustic receptivity experiments\textsuperscript{7–10} attempted to provide validation for computational studies.\textsuperscript{11–14} So far, the experimental evidence has been inconclusive due to sparse data and uncertainty in the Stokes and TS wave separation technique. To provide quality acoustic receptivity measurements, an improved experimental technique must be developed. The method must be efficient so that meaningful amounts of data can be collected and also robust in compensating for or avoiding upstream-traveling reflections and poor frequency resolution.

With these goals in mind, a new method for measuring TS wave amplitudes is proposed that utilizes the biorthogonal properties of a modified OS equation. This technique has several advantages over previous methods, and, combined with active noise control, enables the effective use of continuous forcing to measure leading-edge acoustic receptivity coefficients. The biorthogonal decomposition technique provides order of magnitude efficiency improvement over previous experimental methods by requiring only one boundary-layer scan per receptivity coefficient. Also, acoustic reflections are compensated for using active noise control and the frequency resolution problem is avoided by using continuous forcing. Section II outlines the biorthogonal decomposition technique. Sections III & IV detail its application to experimental data.

II. Modal Decomposition

Tumin et al.\textsuperscript{17} first applied biorthogonal decomposition to experimental data by decomposing the modes of a 2D wall jet. In this method, experimental data is used to find the coefficients of the modes of the OS equation that, when combined, reconstruct the disturbance profile. To compensate for incomplete data, a modal assumption was used for the unknown quantities. Recently, Denissen & White\textsuperscript{18} applied biorthogonal decomposition to experimental measurements of roughness-induced transient growth to compute the amplitude and phases of continuous spectrum modes of the OS equation. In a similar fashion, Stokes waves can be separated from TS waves through modal decomposition.

A. Modified Orr–Sommerfeld Equation

Acoustic disturbances, like TS waves, are small perturbations of the parallel basic state. As such, acoustic disturbances are governed by a modified OS equation. Casting that modified equation such that both TS
waves and acoustic disturbances are solutions enables separation of modes using the direct and adjoint equations. The OS equation governs small fluctuations on an incompressible flow with a parallel basic state. In order for acoustic disturbances to be captured by the OS equation, a small amount of compressibility must be allowed.

Starting from the compressible linearized conservation of mass and momentum equations, the basic state is assumed parallel and incompressible, temperature and viscosity perturbations are ignored, barotropic density perturbations are related to the pressure disturbances by:

$$\rho' = M_\infty^2 p',$$

and the continuity and momentum equations are decoupled from the energy equation.

The modified OS equation has the same form as Eq. (1) but has the modified operators given in the Appendix. This modified formulation of the OS equation is equivalent to the low Mach number limit ($M_\infty \ll 1$) of the fully compressible 2D linear stability equations.

Using zero velocity fluctuation boundary conditions ($u = v = 0$) at the wall and the freestream yields discrete modes only. To admit acoustic disturbance solutions of the modified equations, instead of $\hat{u} = 0$ and $\hat{v} = 0$, the freestream boundary conditions are relaxed and only boundedness is required: $|\hat{u}| < \infty$ and $|\hat{v}| < \infty$ as $y \to \infty$.

In the freestream, the operators $A$ and $B$ reduce to matrices of constant coefficients and solutions take the form $\phi \propto e^{iy}$. Since the modified OS equation is fourth order, there are four asymptotic freestream eigenvalues:

$$\lambda_{1,2} = \pm i \sqrt{F R^2 + i \alpha^2 - R \alpha},$$

$$\lambda_{3,4} = \pm \sqrt{\frac{i F M_\infty^2 R^2 \lambda^2 - R \alpha^2 + F^2 M_\infty^2 R = 1 - i M_\infty^2 \alpha^3 l_2 - 2 F M_\infty^2 R^2 \alpha + M_\infty^2 \alpha^2}{i F M_\infty^2 R l_2 - i M_\infty^2 \alpha l_2 - R}}.$$

where $l_j = j + \lambda$, and from Stokes’ hypothesis $\lambda = -\frac{2}{3}$.

There are two classes of solutions to the modified OS equation: discrete and continuous spectrum modes. Discrete modes, such as TS waves, asymptotically approach zero as $y \to \infty$ and are a linear combination of the eigenvectors associated with $\lambda_2$ and $\lambda_4$, the eigenvalues with negative real part. There are only a finite number of discrete spatial eigenvalues, $\alpha$, such that the boundary conditions at $y = 0$ are satisfied.

Continuous spectrum modes only require boundedness as $y \to \infty$. This requires $\text{Re}(\lambda) \leq 0$ in the freestream. Solving for the spatial eigenvalues, $\alpha$, such that $\lambda = ik$ where $k \in \mathbb{R}$ and $k > 0$, defines the continuous spectrum. Since freestream continuous spectrum solutions have the form $e^{iky}$, $k$ can be interpreted as a wall-normal wavenumber. If $\alpha$ is chosen such that $\lambda_3 = ik$, the continuous spectrum modes are found by a linear combination of the eigenvectors associated with $\lambda_2$, $\lambda_3$, and $\lambda_4$ such that the wall boundary conditions and an arbitrary normalization are satisfied.

Figure 1 shows the two acoustic branches of the continuous spectrum. Stokes waves are the eigenfunctions as $k \to 0^+$ of the upstream and downstream acoustic modes, where the streamwise wavenumber approaches $RF M_\infty/(M_\infty \pm 1)$.

For a given nondimensional frequency ($F$), Reynolds number ($R$), and basic state ($U(y)$), the OS equation is solved by integrating from the freestream to the wall using a Gram-Schmidt orthonormalization procedure. TS waves computed using the standard and modified equations for $M_\infty \ll 1$ produce the same eigenvalue and same mode shape. Computations with the relaxed boundary conditions allow upstream- and downstream-traveling continuous-spectrum acoustic disturbances. Example mode shapes resulting from calculations using the modified OS equation are shown in Figure 2. Because of the small compressibility effects, the streamwise wavenumbers of Stokes waves are consistent with the freestream acoustic wavelength at the forcing frequency.

The adjoint equation and boundary conditions are found by multiplying Eq. (1) by $\psi^T$ and integrating by parts:

$$A \frac{\partial \psi}{\partial y} = -B^T \psi - \frac{\partial A}{\partial y} \psi,$$

$$y = 0: \quad \psi_2 = 0, \quad \psi_4 = 0,$$

$$y \to \infty: \quad |\psi_2| < \infty, \quad |\psi_4| < \infty.$$
Figure 1. Downstream and upstream acoustic branches of the continuous spectrum for $M_\infty = 0.3$, $R = 20$, and $F = 0.05$. The continuous spectrum for a low Reynolds number and relatively high Mach number is shown to exaggerate the effects of viscous damping. Stokes waves are the modes as $k \to 0^+$ of the upstream and downstream acoustic branches.

Figure 2. Example mode shapes for the modified OS equation at $R = 717$, $F = 100 \times 10^{-6}$, and $M_\infty = 0.04$, (a) TS wave, $\alpha = 0.199$, (b) Stokes wave, $\alpha = +0.00276/-0.00299$

B. Biorthogonality Condition

For the derivation of a biorthogonality condition, the matrix operators in Eqs. (1) & (2) need to be factored into quantities independent of $\alpha$. First, the state vector is augmented to remove nonlinearities in $\alpha$:

\[
\tilde{\phi} = \left[ \hat{u}, \frac{\partial \hat{u}}{\partial y}, \hat{v}, \hat{p}, i\alpha \hat{u}, i\alpha \hat{v}, i\alpha \hat{p} \right]^T.
\]
Then the augmented direct and adjoint equations are:

\[
\tilde{A} \frac{\partial \tilde{\phi}}{\partial y} = \tilde{B} \tilde{\phi} \tag{3}
\]

\[
\tilde{A} \frac{\partial \tilde{\psi}}{\partial y} = -\tilde{B}^T \tilde{\psi} - \frac{\partial \tilde{A}}{\partial y} \tilde{\psi} \tag{4}
\]

where the boundary conditions are unchanged and the associated augmented matrix operators are given in the appendix. This allows the augmented operators to be factored:

\[
\tilde{A} = \tilde{A}_1 + i\alpha \tilde{A}_2 \\
\tilde{B} = \tilde{B}_1 + i\alpha \tilde{B}_2.
\]

By taking the integral of \( \tilde{\psi}_{\alpha'} \), the augmented adjoint solution with spatial eigenvalue \( \alpha' \), multiplied by Eq. (3), a biorthogonality condition is derived.\(^{22}\) Using the factorization above, the integral becomes:

\[
\int_0^\infty \tilde{\psi}_{\alpha'}^T (\tilde{A}_1 + i\alpha \tilde{A}_2) \tilde{D} \tilde{\phi}_\alpha \, dy = \int_0^\infty \tilde{\psi}_{\alpha'}^T (\tilde{B}_1 + i\alpha \tilde{B}_2) \tilde{\phi}_\alpha \, dy.
\]

Integrating by parts, using the adjoint equation, Eq. (4), and using the direct and adjoint boundary conditions, the above equation becomes a biorthogonality inner product:

\[
\left\langle \tilde{\phi}_\alpha, \tilde{\psi}_{\alpha'} \right\rangle &= \int_0^\infty \left[ D \tilde{\psi}_{\alpha'}^T \tilde{A}_2 + \tilde{\psi}_{\alpha'}^T (\tilde{B}_2 + D \tilde{A}_2) \right] \tilde{\phi}_\alpha \, dy = Q_\alpha \delta(\alpha - \alpha'). \tag{5}
\]

In Eq. (5), \( \delta \) is a Dirac delta if \( \tilde{\phi}_\alpha \) and \( \tilde{\psi}_{\alpha'} \) are continuous spectrum modes and \( \delta \) is a Kronecker delta if either is a discrete mode.

In the case of discrete modes, the normalization constant, \( Q_\alpha \), is found by numerically calculating the biorthogonal inner product, Eq. (5). In the case of continuous spectrum modes, more care is needed. Following Tumin,\(^{22}\) the biorthogonal inner product over a narrow wave packet is expanded into integrals in the boundary layer and freestream:

\[
\lim_{\epsilon \to 0} \int_{k-\epsilon}^{k+\epsilon} \left\langle \tilde{\phi}_\alpha, \tilde{\psi}_{\alpha'} \right\rangle \, dk = \lim_{\epsilon \to 0} \int_{k-\epsilon}^{k+\epsilon} \left\langle \tilde{\phi}_\alpha, \tilde{\psi}_{\alpha'} \right\rangle_{\epsilon} \, dk \\
= -\lim_{\epsilon \to 0} \int_{k-\epsilon}^{k+\epsilon} \left\langle \tilde{\phi}_\alpha, \tilde{\psi}_{\alpha'} \right\rangle_{\epsilon} \, dk + \lim_{\epsilon \to 0} \int_{k-\epsilon}^{k+\epsilon} \left\langle \tilde{\phi}_\alpha, \tilde{\psi}_{\alpha'} \right\rangle \, dk \tag{6}
\]

where \( \tilde{\phi}_\alpha \) and \( \tilde{\psi}_{\alpha'} \) are continuous spectrum solutions of the direct and adjoint equations for eigenvalues \( \alpha(k) \) and \( \alpha'(k') \), respectively. The superscript \( \infty \) indicates asymptotic solutions in the freestream and \( L \) is the edge of the boundary layer. The first two integrals on the right side vanish as \( \epsilon \to 0 \) because the integrands are finite. The asymptotic solutions can be written in terms of the freestream eigenvectors of Eq. (3):

\[
\tilde{\phi}_\alpha^\infty(y) = \zeta_3 \tilde{\phi}_3^\infty e^{\lambda_3 y} + \zeta_4 \tilde{\phi}_4^\infty e^{\lambda_4 y} \\
\tilde{\psi}_{\alpha'}^\infty(y) = \zeta_3 \tilde{\psi}_3^\infty e^{\lambda_3 y} + \zeta_4 \tilde{\psi}_4^\infty e^{\lambda_4 y} \tag{7}
\]

where \( \zeta_j \) and \( \xi_j \) are the coefficients of the freestream eigenvectors (\( \tilde{\phi}_j^\infty \) and \( \tilde{\psi}_j^\infty \)) so that the boundary conditions are satisfied. From the definition of the continuous spectrum, \( \lambda_3 = -\lambda_4 = i k \). The last term on the right of Eq. (6) does not vanish and consists of integrals of the type:\(^{22}\)

\[
\lim_{\epsilon \to 0} \int_{k-\epsilon}^{k+\epsilon} \int_0^\infty e^{i(k-k') y} dy \, dk = \pi \delta(k-k') \tag{8}
\]

where \( \delta \) is the Dirac delta. \( Q_\alpha \) for continuous spectrum modes is then known explicitly:

\[
Q_\alpha = \pi M_{ij} (\zeta_3 \tilde{\phi}_j^\infty \xi_3 \tilde{\psi}_i^\infty + \zeta_4 \tilde{\phi}_4^\infty \xi_4 \tilde{\psi}_4^\infty) \tag{9}
\]
where $M = -\mathbf{B} \mathbf{A}^{-1} \mathbf{A}_2 + \mathbf{B}_2$, subscript $i$ and $j$ denote the component of the freestream eigenvectors, and summation is implied.

If the amplitudes of discrete modes and amplitude curves of continuous spectrum modes are known, the complete disturbance profile can be reconstructed:

$$\phi(y) = \sum_d C_d \phi_{\alpha_d}(y) + \sum_j \int_0^\infty C_j(k) \phi_{\alpha_j}(y) dk$$

(10)

where indices $d$ and $j$ are for the discrete modes and branches of the continuous spectrum, respectfully.

C. Experimental Decomposition Method

Downstream- and upstream-traveling Stokes waves are practically inseparable when only the streamwise velocity disturbance at a single $x$ location is known. In this section only downstream-traveling Stokes waves are considered. Since Stokes waves and TS waves are both solutions to the modified OS equation, their amplitudes can be extracted through the biorthogonality condition:

$$C_{TS} = \frac{1}{Q_{TS}} \left\langle \hat{\phi}, \hat{\psi}_{TS} \right\rangle_0^\infty = \frac{\left\langle \hat{\phi}, \hat{\psi}_{TS} \right\rangle_0^\infty}{\left\langle \hat{\phi}_{\text{TS}}, \hat{\psi}_{TS} \right\rangle_0^\infty}$$

(11)

$$\lim_{\epsilon \to 0^+} C_{St} Q_{St} \delta(k - \epsilon) = C_{St} \left( \left\langle \hat{\phi}_{St}, \hat{\psi}_{St} \right\rangle_0^\infty = \left\langle \hat{\phi}, \hat{\psi}_{St} \right\rangle_0^\infty \right)$$

(12)

where $\hat{\phi}$ is the full set of velocity and pressure data, $\hat{\psi}_{TS}$ is the TS wave solution to the adjoint modified OS equation, and $\hat{\psi}_{St}$ is the Stokes wave solution of the adjoint modified OS equation. Note that $C_{St}$ is related to the continuous spectrum amplitude curve, defined in Eq. (10), as $C_{St}(k) = \lim_{\epsilon \to 0^+} C_{St} \delta(k - \epsilon)$. For the experimental decomposition method, $Q_{St}$ in Eq. (12) above is left in its more general definition:

$$\lim_{\epsilon \to 0^+} Q_{St} \delta(k - \epsilon) = \left\langle \hat{\phi}_{St}, \hat{\psi}_{St} \right\rangle_0^\infty.$$  

(13)

The only component of $\hat{\phi}$ that can be measured reliably in an experiment is the streamwise component $\hat{u}$. Tumin et al.\(^{17}\) used a superposition of two mode shapes to assume a form for the unknown quantities in $\hat{\phi}$. For the following modal assumption is made:

$$\hat{\phi} = \hat{\phi}_{\text{exp}} + C_{TS} \hat{\phi}_{TS}^{(0)} + \lim_{\epsilon \to 0^+} C_{St} \hat{\phi}_{St}^{(0)} \delta(k - \epsilon) dk = \hat{\phi}_{\text{exp}} + C_{TS} \hat{\phi}_{TS}^{(0)} + C_{St} \hat{\phi}_{St}^{(0)}$$

(14)

where $\hat{\phi}_{\text{exp}}$ contains the experimentally measured streamwise velocity disturbance with the unmeasurable components set to zero. $\hat{\phi}_{TS}^{(0)}$ and $\hat{\phi}_{St}^{(0)}$ are the TS and Stokes wave solutions with the first component set to zero corresponding to measurements in $\hat{\phi}_{\text{exp}}$. Only these two modes are included because downstream-traveling sound dominates the freestream disturbance environment and TS waves are the only unstable mode. Upstream-traveling acoustic modes are minimized with active noise control and vortical freestream disturbances are orders of magnitude smaller than the downstream-traveling acoustic disturbance. Substituting this modal assumption into Eqs. (11) & (12), a linear system of equations is found for the complex amplitude of Stokes and TS waves:

$$\begin{bmatrix}
Q_{TS} & \left\langle \hat{\phi}_{TS}^{(0)}, \hat{\psi}_{TS} \right\rangle_0^\infty \\
-\left\langle \hat{\phi}_{TS}^{(0)}, \hat{\psi}_{St} \right\rangle_0^\infty & \left\langle \hat{\phi}_{St}^{(0)}, \hat{\psi}_{St} \right\rangle_0^\infty \\
\lim_{\epsilon \to 0^+} C_{St} Q_{St} \delta(k - \epsilon) & \left\langle \hat{\phi}_{St}^{(0)}, \hat{\psi}_{St} \right\rangle_0^\infty
\end{bmatrix}
\begin{bmatrix}
C_{TS} \\
C_{St}
\end{bmatrix}
= \begin{bmatrix}
\left\langle \hat{\phi}_{\text{exp}}, \hat{\psi}_{TS} \right\rangle_0^\infty \\
\left\langle \hat{\phi}_{\text{exp}}, \hat{\psi}_{St} \right\rangle_0^\infty
\end{bmatrix}.$$  

(15)

If the term with $Q_{St}$ is replaced by Eq. (13), this term becomes an integral over a finite domain:

$$\lim_{\epsilon \to 0^+} Q_{St} \delta(k - \epsilon) = \left\langle \hat{\phi}_{St}^{(0)}, \hat{\psi}_{St} \right\rangle_0^\infty = \left\langle \hat{\phi}_{St}^{(1)}, \hat{\psi}_{St} \right\rangle_{y_{\min}}^{y_{\max}}$$

(16)
using two sets of microphones at the same streamwise but different spanwise locations. For this measurement, downstream-traveling sound, for upstream- and downstream-traveling sound waves that were decomposed frequencies at the tunnel test conditions. Figure 3 shows the sound pressure level, relative to the baseline descent algorithm continuously updates the FIR filter such that the upstream-traveling sound is minimized. locations in the test section separate upstream-traveling and downstream-traveling sound and a gradient secondary subwoofers in order to cancel upstream-traveling waves. Four microphones at different streamwise filter is used to filter the signal sent to the primary subwoofers; the filtered signal is used to drive the sound waves that cancel the diffuser reflection using closed-loop control. A finite impulse response (FIR) strategy developed by Kuester & White.

leading edge that cannot be separated from the TS waves created by downstream-traveling sound. Thus the upstream-traveling reflected waves generate TS waves at the very long wavelengths. For the present experiment, the acoustic wavelength is the same order of magnitude and a frequency selection effect.

One of the major challenges involved with continuous acoustic forcing is the complex duct acoustics. Continuous forcing can distort receptivity measurements by creating a standing wave pattern in the test section due to a reflection from the downstream diffuser. The reflection from the diffuser can also cause wake resonance and a frequency selection effect. Hammerton & Kerschen showed that the leading edge is much more receptive to upstream-traveling waves than downstream for very short wavelengths and is equally receptive for very long wavelengths. For the present experiment, the acoustic wavelength is the same order of magnitude as the semichord of the flat plate. Thus the upstream-traveling reflected waves generate TS waves at the leading edge that cannot be separated from the TS waves created by downstream-traveling sound.

To enable the use of continuous forcing, the diffuser reflection is eliminated using an active noise control strategy developed by Kuester & White. Two secondary subwoofers downstream of the test section emit sound waves that cancel the diffuser reflection using closed-loop control. A finite impulse response (FIR) filter is used to filter the signal sent to the primary subwoofers; the filtered signal is used to drive the secondary subwoofers in order to cancel upstream-traveling waves. Four microphones at different streamwise locations in the test section separate upstream-traveling and downstream-traveling sound and a gradient descent algorithm continuously updates the FIR filter such that the upstream-traveling sound is minimized.

Before performing the receptivity experiments, the active noise control system was tested at multiple frequencies. The system of equations now becomes:

\[
\begin{bmatrix}
Q_{TS} - \langle \tilde{\phi}_{TS}^{(0)} , \tilde{\psi}_{TS}\rangle|_{0}^{\infty} - \langle \tilde{\phi}_{TS}^{(1)} , \tilde{\psi}_{St}\rangle|_{y_{min}}^{y_{max}} \\
C_{TS} - \langle \tilde{\phi}_{St}^{(0)} , \tilde{\psi}_{St}\rangle|_{0}^{\infty} - \langle \tilde{\phi}_{St}^{(1)} , \tilde{\psi}_{St}\rangle|_{y_{min}}^{y_{max}} 
\end{bmatrix}
= \begin{bmatrix}
\langle \tilde{\phi}_{exp} , \tilde{\psi}_{TS}\rangle|_{y_{min}}^{y_{max}} \\
\langle \tilde{\phi}_{exp} , \tilde{\psi}_{St}\rangle|_{y_{min}}^{y_{max}} 
\end{bmatrix} \tag{17}
\]

and all integrals are convergent since direct and adjoint TS waves approach zero exponentially as \( y \to \infty \).

The limits, \( y_{max} \) and \( y_{min} \), are determined by the capabilities of the experimental setup. For this experiment the boundary-layer scans start at approximately twice the boundary-layer thickness, or \( y_{max} \approx 10 \), and the minimum \( y \) limit corresponds to \( y^+ = 4 \), or \( y_{min} = 4/\sqrt{R(\partial U/\partial y)|_{y=0}} \), which is the minimum distance at which wall-effects are insignificant for hotwire anemometry. This limit captures the maximum of the TS wave and numerical experiments show it is sufficient for accurate decomposition.

The modal decomposition technique has several advantages over the complex-plane spiral analysis and pulsed-sound decomposition methods. First, because it only needs data from a single streamwise location, the TS wave amplitude can be measured quickly relative to previous techniques which require multiple boundary-layer scans to obtain just one receptivity coefficient. Second, because continuous forcing is used, this technique does not have the frequency resolution, speed, and data quality problems associated with the pulsed-sound technique.

III. Experimental Facility & Approach

The receptivity measurements were performed in the Klebanoff–Saric Wind Tunnel (KSWT) at Texas A&M University. The KSWT is a low-disturbance, low-speed wind tunnel specifically designed for boundary-layer stability and transition work. For the test conditions, the background \( u’_{rms} \) fluctuations are approximately 0.02% spread over a broad range of frequencies. The acoustic forcing is the same order of magnitude, but is localized to a single frequency and dominates the disturbance environment. A complete description of the tunnel, including flow quality, is given by Hunt et al. During construction, broadband acoustic panels and acoustic foam were added inside the tunnel to lower background flow noise from the fan and motor. These acoustic treatments were chosen specifically to reduce noise in the TS passband. Sound is introduced using five 10-inch diameter subwoofers mounted upstream of the test section. The subwoofers emit near-planar sound that interacts with the model to generate TS waves. To prevent nonlinear effects, the sound amplitude is limited to 100 dB in the test section.

A. Active Noise Control

One of the major challenges involved with continuous acoustic forcing is the complex duct acoustics. Continuous forcing can distort receptivity measurements by creating a standing wave pattern in the test section due to a reflection from the downstream diffuser. The reflection from the diffuser can also cause wake resonance and a frequency selection effect. Hammerton & Kerschen showed that the leading edge is much more receptive to upstream-traveling waves than downstream for very short wavelengths and is equally receptive for very long wavelengths. For the present experiment, the acoustic wavelength is the same order of magnitude as the semichord of the flat plate. Thus the upstream-traveling reflected waves generate TS waves at the leading edge that cannot be separated from the TS waves created by downstream-traveling sound.

To enable the use of continuous forcing, the diffuser reflection is eliminated using an active noise control strategy developed by Kuester & White. Two secondary subwoofers downstream of the test section emit sound waves that cancel the diffuser reflection using closed-loop control. A finite impulse response (FIR) filter is used to filter the signal sent to the primary subwoofers; the filtered signal is used to drive the secondary subwoofers in order to cancel upstream-traveling waves. Four microphones at different streamwise locations in the test section separate upstream-traveling and downstream-traveling sound and a gradient descent algorithm continuously updates the FIR filter such that the upstream-traveling sound is minimized.

Before performing the receptivity experiments, the active noise control system was tested at multiple frequencies at the tunnel test conditions. Figure 3 shows the sound pressure level, relative to the baseline downstream-traveling sound, for upstream- and downstream-traveling sound waves that were decomposed using two sets of microphones at the same streamwise but different spanwise locations. For this measurement,
the speaker wall was excited at a constant forcing amplitude. Upstream-traveling wave amplitudes are 30 dB less than downstream-traveling waves on average. Thus, using active noise control greatly reduces contamination from acoustic reflections during continuous forcing and allows modal decomposition of the boundary-layer disturbances.

Figure 3. Upstream-traveling wave amplitudes are 30 dB less than downstream-traveling waves on average using active noise control. Blue dashed lines indicate upstream-traveling waves and black lines indicate downstream-traveling waves. Squares indicate the microphone set at spanwise location 1 and circles indicate the microphone set at location 2.

B. Flat-Plate Model

The model used is an aluminum-nickel alloy flat plate with a 20:1 modified-super-ellipse (MSE) leading edge. This is the same model used by Saric et al., Saric & White, and White et al. in previous experiments. The plate is 9.53 mm thick, 4000 mm long, and 1370 mm wide. The leading edge is located approximately 690 mm downstream of the contraction exit plane. The MSE leading edge eliminates the discontinuity in surface curvature at the leading-edge/flat-plate juncture, which is a possible receptivity location. To further ensure that the interface is smooth, the leading edge is machined directly on the flat plate.

The flat plate is mounted so that the streamwise direction is horizontal and the spanwise direction is vertical in the test section. Five adjustable mounting brackets allow for fine alignment to ensure $\partial P/\partial x = 0$. Following the recommendation by Klewicki et al., the flat plate is aligned so that $H = \delta^*/\theta = 2.591 \pm 0.005$. Plate alignment is crucial to ensure that the $N$-factor growth rates match linear stability theory for Blasius flow. A 500 mm trailing-edge flap is used to adjust the location of the stagnation line. Differential pressure sensors on either side of the leading edge are used to ensure symmetry about the leading edge. A trip strip located on the non-test side of the plate fixes the transition location and ensures constant blockage.

Due to non-zero pressure gradients near the leading-edge, the streamwise coordinate is referenced to the virtual leading edge, $\bar{x}_{vle}$. The virtual leading edge is found through a least squares fit to the growth of the experimentally measured momentum thickness to be $55 \pm 9$ mm downstream of the physical leading edge.
C. Receptivity Measurement Technique

To perform a receptivity measurement for a particular F and plate-half-thickness Reynolds number, a wall normal hotwire scan is performed at the branch II location on the flat plate. The TS wave is measured at the branch II location so that it grows to maximum amplitude and provides the best possible signal-to-noise ratio. All measurements are taken with a constant Reynolds number based on the plate-half-thickness, $R_b = 2430$, to match previous computational studies.\textsuperscript{13, 14} The complex amplitude $\hat{u}$ at the forcing frequency is found using an FFT with a flat-top window function of the time-varying data.

After determining the amplitude of the TS wave through modal decomposition, the receptivity coefficient is defined by scaling the amplitude to its branch I value and dividing by the amplitude of the freestream sound wave at the leading edge:\textsuperscript{7}

$$K_I = \frac{\left| \hat{u}_{TS} \right|_{\text{Branch I}}}{\left| \hat{u}_{St} \right|_{\text{LE}}} = \frac{\left| \hat{u}_{TS} \right|_{\text{Branch II}}}{e^N \left| \hat{u}_{St} \right|_{\text{LE}}}. \quad (18)$$

The issue of non-parallel acoustic disturbances naturally arises since the acoustic wavelength can be very large. The decomposition method of Eq. (17) is derived assuming parallel flow and does not accommodate non-parallel disturbances. Ackerberg & Phillips\textsuperscript{27} found solutions to the unsteady boundary layer equations with small time-periodic fluctuations in the freestream. They found a similarity parameter $\xi = FR^2$ (the authors use the variable $x$ instead of $\xi$) which is related to the degree of non-parallelism. As $\xi \to \infty$, the disturbance converges to an analytical Stokes wave. Figure 4 shows two comparisons between analytical Stokes waves and solutions of the unsteady boundary layer equations for $\xi = 1$ and $\xi = 4$. For $\xi > 4$ the disturbances are practically equivalent. The neutral stability curve, contours of constant $\xi$, and the experimental test parameters are shown in Figure 5. The neutral stability curve was computed by numerically solving Eq. (1). The minimum value of $\xi$ which is encountered in the experiment is $\xi = 49$ and as a result non-parallel acoustic disturbances are not an issue.

![Figure 4](image)

Figure 4. Comparison of analytical Stokes waves to solutions of the unsteady boundary layer equations, (a) $\xi = 1$, (b) $\xi = 4$.

IV. Results

The decomposition method developed in section II is used to extract amplitudes of the TS and Stokes waves from boundary-layer scans of the present experiment. One check of the decomposition accuracy is to compare the reconstructed velocity field with the measured one.\textsuperscript{22} Figure 6 shows the reconstruction of experimental data using biorthogonal decomposition results for the nondimensional frequency, $F = 77.1 \times 10^{-6}$, and Reynolds number, $R = 843$. The reconstruction quality of this plot is representative of each decompo-
Figure 5. Neutral stability curve for Blasius flow overlaid with the frequency range for which acoustic receptivity is measured and contours of constant $\xi$. The neutral stability curve was computed by numerically solving Eq. (1).

sition included in the receptivity results. The experimental decomposition method accurately reconstructs the fluctuating boundary-layer profile using only downstream-traveling Stokes and TS waves.

Because the receptivity coefficient is measured at the branch II location and scaled to branch I using linear stability theory, it is imperative that the streamwise growth of TS waves matches linear stability theory. To verify this, tape was placed at the branch I location on the plate and the speaker wall/tunnel were set to achieve a nondimensional frequency of $F = (56.8 \pm 0.2) \times 10^{-6}$ with the goal of generating easily measurable TS waves. TS wave amplitudes were extracted using the biorthogonal decomposition method outlined in section II. These experimentally measured TS wave amplitudes are compared to what linear stability theory predicts in Figure 7. The largest measured TS wave amplitude is around 1.4% of the freestream velocity and is likely in the first stages of breakdown. The results confirm that the setup provides growth rates consistent with linear theory and provides confidence that measurements at branch II give accurate $K_I$ results.

In Figure 8, the growth of TS waves generated by the acoustic receptivity process at the leading edge, without the tape present, is compared to linear stability theory. The error bars are the standard error of the TS wave amplitudes. Again, the TS wave amplitudes are extracted using the biorthogonal decomposition method. There is more spread in the data due to substantially lower TS wave amplitudes. Nevertheless, the agreement remains quite good.

Figure 9 displays the experimentally measured acoustic receptivity coefficients for a 20:1 MSE leading edge using both biorthogonal decomposition and pulsed-sound methods along with the DNS results of Wanderley & Corke\textsuperscript{14} and Fuciarelli & Reed\textsuperscript{13} for the same geometry and Reynolds number. For comparison, the results of Saric & White\textsuperscript{8} and new pulsed sound results are presented. Shown are 50 receptivity coefficients obtained using biorthogonal decomposition. Although there is large spread in the data, the acoustic receptivity results are consistent with the DNS results of Wanderley & Corke\textsuperscript{14} and Fuciarelli & Reed.\textsuperscript{13} The spread is likely due to small TS wave amplitudes based on the differences between Figs. 7 & 8. Unlike previous experimental studies,\textsuperscript{7–10} the receptivity coefficients agree with the computations for a broad range of frequencies.
An additional 106 coefficients were measured and computed but have been omitted from this plot. A weighted residual, $\hat{e}$, was computed for each reconstruction.

$$e = \phi_{\text{exp}} - \left( C_{\text{TS}} \tilde{\phi}_{\text{TS}}^{(1)} + C_{\text{St}} \tilde{\phi}_{\text{St}}^{(1)} \right)$$

$$\hat{e} = \frac{\int_0^\infty |e \cdot \tilde{\phi}_{\text{TS}}^{(1)}| dy}{\|C_{\text{TS}} \tilde{\phi}_{\text{TS}}^{(1)} + C_{\text{St}} \tilde{\phi}_{\text{St}}^{(1)}\| \int_0^\infty |\tilde{\phi}_{\text{TS}}^{(1)}| dy}$$

The inclusion criteria for figure 9 was $\hat{e} < 0.18$. Receptivity coefficients not meeting this criteria have poor reconstructions most likely due to the presence of additional acoustic modes that are unaccounted for in the decomposition. Subsequent to this work, significant structural vibrations were measured in the test section near 57 and 65 Hz. Results near these frequencies have also been omitted from figure 9. Still, the amount of data obtained in the present experiment is orders of magnitude larger than all previous acoustic receptivity experiments.

To provide additional comparison between the current and previous results, pulsed-sound measurements were taken for the same model and flow conditions. These measurements followed the methods of Saric & White and White et al. The receptivity coefficients obtained using the pulsed-sound method are compared to the results using biorthogonal decomposition in the next section.

Figure 10 shows a time-trace of the voltages for both the boundary-layer and freestream hotwires during pulsed forcing. From this plot, it is clear that the pulsed-sound method effectively separates the Stokes and TS waves. However, measurement of the receptivity coefficient from this data is ambiguous. The function generator inputs a 4-cycle pulse but the freestream hotwire measures a 10-cycle acoustic wave packet. The boundary-layer hotwire measures a much longer Stokes wave and, after a short delay, a 14-cycle TS wave pulse. Depending on the width of the sampling window for the Fourier transform, the receptivity coefficient amplitude can change significantly. It is not clear if this spreading is due to wave dispersion or from receptivity to acoustic reflections. For these results, a 4-cycle sampling window is used for both the TS wave signal and the freestream Stokes wave signal.

Figure 9 includes two pulsed-sound data points from the present experiment. They are of the same order of magnitude as both the DNS and biorthogonal decomposition results. The fact that they are significantly lower than both the DNS and biorthogonal decomposition results is not surprising due to the sampling window issue mentioned previously. If the sampling window is increased beyond the 4-cycle input, the resulting receptivity coefficients are much closer to the biorthogonal decomposition results. Many hours of measurements were required to obtain these two data points as opposed to the minutes required to obtain a receptivity coefficient using the biorthogonal decomposition method.

To match receptivity coefficients defined at branch I to theoretical acoustic receptivity at the leading edge, Saric et al. performed linear stability calculations on a basic state determined by DNS for a flat plate with a 20:1 MSE leading edge. For $F = 84 \times 10^{-6}$ and $R_b = 2270$, this calculation resulted in an amplitude ratio decay of 0.154 from $x \approx \lambda_{\text{TS}} / 2$ to the branch I location. Using this amplitude ratio, a receptivity coefficient from the present work scaled to the leading edge is found to be $K_{\text{LE}} = 0.3$. This is the same order of magnitude as the theoretical calculation of $K_{\text{LE}} = 0.95$ by Kerschen et al. and the finite-nose-radius calculations of $K_{\text{LE}} = 0.48$ for $St = 0.01$ by Haddad & Corke.

V. Conclusions

Boundary-layer receptivity to freestream acoustic disturbances is fundamental to the origin of TS waves. Past experiments have addressed acoustic receptivity using continuous forcing and pulsed sound, both of which pose experimental difficulties. In this paper, an experimental biorthogonal decomposition method combined with active noise control addresses the issues with continuous forcing and enables its use. The experimental decomposition algorithm successfully measures receptivity coefficients much more efficiently than previously possible.

The receptivity coefficients obtained through biorthogonal decomposition are in reasonable agreement with those found using the pulsed sound method, especially when the pulsed-sound sampling window ambiguity is taken into account. The measured acoustic receptivity data is consistent with the DNS results of both Wanderley & Corke and Fuciarelli & Reed. Looking at the quality of the individual decompositions, it is clear that the experimental biorthogonal decomposition method is able to accurately reconst...
the experimental fluctuating boundary-layer measurements using only TS and downstream-traveling Stokes waves.

Using the amplitude ratio decay calculated by Saric et al.,\(^7\) an approximate leading-edge receptivity coefficient, \(K_{LE} = 0.3\), was found. This value compares well to theory by Kerschen et al.\(^6\) and calculations by Haddad & Corke.\(^3\)

The biorthogonal decomposition method is a powerful method to separate Stokes and TS waves. The biorthogonality condition is not only applicable in experiments, but could also be utilized in future DNS acoustic receptivity studies to separate the continuous spectrum acoustic disturbances and TS waves.

The acoustic receptivity of certain leading edge geometries has been studied extensively, but these measurements need to be extended to more practical flow fields. For example, receptivity theory\(^15\) and computations\(^31\) have analyzed the receptivity of parabolic leading edges at non-zero angles of attack, but these calculations have not been validated by experiments. Future work will extend the decomposition method to the experimental studies of more complex geometries.

Appendix

The Modified Orr–Sommerfeld (OS) operators for both the compact and augmented equations are given below. The conventional OS operators are obtained by setting \(M_\infty = 0\). The Stokes’ hypothesis is used for the present work, \(\lambda = -\frac{2}{3}\), and \(l_j = j + \lambda\).

\[
A = \text{diag} \left( 1, 1, 1 + \frac{iM_\infty^2 l_2}{R} (\alpha U - RF) \right)
\]

\[
B = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
\rho R \frac{\partial U}{\partial y} & 0 & iR\alpha - \alpha M_\infty^2 (\alpha U - RF) l_1 \\
-\alpha & 0 & 0 & -iM_\infty^2 (\alpha U - RF) \\
0 & -\frac{i\alpha}{R} & -i(\alpha U - RF) - \alpha^2 & -\frac{i\alpha}{R} M_\infty^2 \frac{\partial U}{\partial y} l_2 \\
\end{bmatrix}
\]

\[
\tilde{A} = \text{diag} \left( 1, 1, 1 + \frac{iM_\infty^2 l_2}{R} (\alpha U - RF) , 0 , 0 , 0 \right)
\]

\[
\tilde{B} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\rho R \frac{\partial U}{\partial y} & 0 & iR\alpha + \alpha M_\infty^2 RF l_1 & -\alpha & 0 & i\alpha M_\infty^2 U l_1 \\
-\alpha & 0 & 0 & -iM_\infty^2 (\alpha U - RF) & 0 & 0 & 0 \\
0 & -\frac{i\alpha}{R} & -i(\alpha U - RF) & -\frac{i\alpha}{R} M_\infty^2 \frac{\partial U}{\partial y} l_2 & 0 & \frac{i\alpha}{R} & 0 \\
\alpha & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & i\alpha & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & i\alpha & 0 & 0 & -1 \\
\end{bmatrix}
\]

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Figure 6. Reconstruction of data from experimental decomposition for $F = 77.1 \times 10^{-6}$ and $R = 843$. The weighted residual, $\hat{e}$, is 0.12. (a) real component of the streamwise velocity disturbance (b) imaginary component of the streamwise velocity disturbance.
Figure 7. Biorthogonal decomposition of the growth of TS waves generated by the interaction of acoustic disturbances and tape placed at the branch I location for \( F = (56.8 \pm 0.2) \times 10^{-6} \) compared with linear stability theory.
Figure 8. Biorthogonal decomposition of the growth of TS waves, generated by leading-edge acoustic receptivity, for $\mathcal{F} = (60.1 \pm 0.2) \times 10^{-6}$ compared with linear stability theory. Error bars are the standard error of TS wave amplitudes.
Figure 9. Acoustic receptivity coefficients, experimentally measured and decomposed using biorthogonal decomposition. Comparison is made to new pulsed sound measurements and those of Saric & White and to the acoustic receptivity coefficients obtained through DNS by Wanderley & Corke and Fuciarelli & Reed.
Figure 10. Time-trace of voltages for both the boundary-layer and freestream hotwires using the pulsed-sound method for $F = 83.2 \times 10^{-6}$. 